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Nematic Liquid Crystals as a Tabletop Platform for Studying Turbulence

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Abstract

Light propagating in nonlinear optical materials opens the possibility to emulate quantum fluids of light with accessible tabletop experiments by taking advantage of the hydrodynamical interpretation. In this context, various optical materials have been studied in recent years, with nematic liquid crystals appearing as one of the most promising ones due to their controllable properties. Indeed, the application of an external electric field can tune their nonlocal response, and this mechanism may be useful for producing fluids of light and developing optical analogues. In this work, we extend the applicability of nematic liquid crystal to support optical analogues and study the possibility of emulating turbulent phenomena by using two fluids of light. These fluids interact with each other through the nonlinearity of the medium and generate instabilities that will lead to turbulent regimes. We also explore the possibility of exciting turbulent regimes through the decay of dark soliton stripes. The preliminary results are presented.

Author Keywords. Nematic Liquid Crystals, Turbulence, Optical Analogues, Fluids of Light, GPGPU Supercomputing.

Type: Research Article

1. Introduction

A light beam propagating in a nonlinear material can be interpreted as a fluid, where the nonlinearity mediates the required interactions and the diffraction in the transverse plane gives the necessary effective mass to the photons. In these systems, the fluid dynamics occur in the transverse plane and the propagation direction plays the role of an effective time, where different slices along the propagation direction represent different moments in time of the fluid (Carusotto 2014; Larré and Carusotto 2015). Different optical materials have been considered for implementing these fluids of light, however, Nematic Liquid Crystals (NLCs) have recently been proposed as potential candidates (Ferreira, Silva, and Guerreiro 2018), since they offer interesting opportunities in the current context.

In short, NLCs are a nonlocal material where the nonlinearity depends on the relative alignment between the orientation of the constituent molecules and the polarization of the beam, and by applying a quasi-static electric field perpendicular to the propagation direction, it is possible to tune its nonlinear response (Hu et al. 2006; Peccianti and Assanto 2012). In this work, we explore the capacity of using fluids of light created with Nematic Liquid crystals to support optical analogues of turbulence. The interest in using these fluids of light, in the present case supported by NLCs, for generating turbulence is that they offer the opportunity to generate and study turbulent regimes in a realistic 2-dimensional setup, instead of the approximated systems that are usually considered. This 2D configuration introduces the conservation of enstrophy, a quantity related to the vorticity of the fluid, that allows the existence of new turbulent regimes (Horng et al. 2009), and their understanding are currently an active topic of discussion (Koniakhin et al. 2020). For exploring the possibility of generating these turbulent states with fluids of light in NLCs, we consider the interaction between two fluids of light with different velocities (Silva, Ferreira, and Guerreiro 2021a). From this interaction, instabilities will develop and will generate turbulence in the system. Indeed, turbulence is a manifestation of these hydrodynamic instabilities and can be characterized by the energy cascades in the incompressible component of the kinetic energy (Kolmakov, McClintock, and Nazarenko 2014). These cascades feature different power laws, and their understanding is still widely discussed. In this work, we report the preliminary results for exciting a turbulent regime through the two-fluid method with a specific zone of energy injection. Furthermore, we also present the preliminary results of a different method for generating turbulent states with these systems, which consists in observing the decay of dark soliton stripes. During the propagation inside the NLC cell, these dark soliton stripes will decay into vortex pairs and will generate a turbulent state. Through both methods, we show that distinct turbulent states may be produced, and their energy cascades can be observed. All the results presented in this work were obtained through numerical solvers of the nonlinear Schrödinger equation based on GPGPU supercomputing developed at our research group (Ferreira, Silva, and Guerreiro 2019; Ferreira et al. 2020; Ferreira et al. 2021; Silva, Ferreira, and Guerreiro 2021b, 2021c).

2. Physical Model

Let us consider two light beams with the same wavenumber, E_1 and E_2, that propagate inside a defocusing NLC cell. Under the paraxial approximation, the overall system is described by the following set of equations (Skuse and Smyth 2009; Izdebskaya et al. 2016).

$$\begin{gathered} i\frac{\partial E_1}{\partial z}+\frac{1}{2}\nabla^2 E_1-2\theta E_1=0 \\ i\frac{\partial E_2}{\partial z}+\frac{1}{2}\nabla^2 E_2-2\theta E_2=0 \\ \nu\nabla^2\theta-2q\theta=-2|E_1|^2-2|E_2|^2, \end{gathered} \tag{1}$$

where ν is related to the nonlocal character of the medium and q is proportional to the applied external electric field, which is used to tune the nonlinear properties of the system. The field θ is related with the small deviations of the constituent molecules in relation to the equilibrium position and represents the nonlinear and nonlocal response of the system. These two light beams will create the two interacting fluids of light, and to simplify we consider that these two fluids of light have equal masses, meaning that $E_1 = E_2 = \sqrt{\rho}$. Furthermore, we also consider that E_1 is stationary and E_2 moves with velocity $\vec{v} = v\hat{x}$ in the transverse plane, where this velocity can be tuned by controlling the angle of entry of the light beam E_2 into the NLC cell. For the dark soliton stripes scenario, only one light beam, or fluid of light, is required, and thus Equation (1) is simplified by making $E_2 = 0$.

3. Results

The dispersion relation of the system with two fluids of light can be calculated by linearizing the hydrodynamic equations that are obtained by applying the Madelung transformation to Equation (1) (Madelung 1927). The result is plotted in Figure 1, and this dispersion relation is composed of two branches that depend on the velocity v. Indeed, by controlling this velocity, it is possible to choose different ranges of instability in one of the branches that will generate different types of turbulence with different energy injection zones.

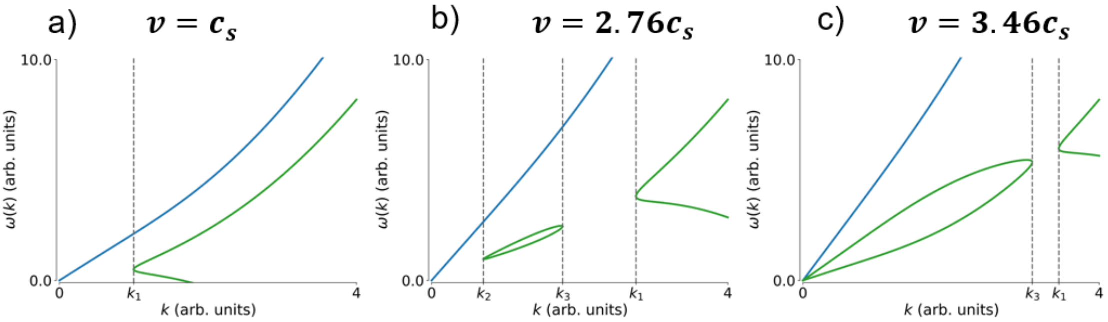

Figure 1: Dispersion relation for the two light fluids system for different velocities of the field E_2. The parameter $c_s = \sqrt{2\rho/q}$ is the sound velocity of the fluid of light. These results were obtained with $\nu = 1$, $q = 2$ and $\rho = 1$

In Figure 2 is plotted the results of a simulation of the two-fluid system with a velocity $v = 3.46c_s$, corresponding to the dispersion relation plotted in Figure 1-c). In Figure 2-a) and Figure 2-b) the density plots of both fluids are shown, and in Figure 2-c) the incompressible component of the kinetic energy is plotted. From this last figure, we see that the system exhibits two energy cascades separated by the energy injection range delimited by the k_i values. Below the energy injection, $k < k_3$, we have an energy cascade that follows the $k^{-5/3}$ power law and above the energy injection, $k > k_1$, we have the enstrophy cascade that follows the k^{-4} power law (Horng et al. 2009). These power laws are a characteristic of turbulent regimes.

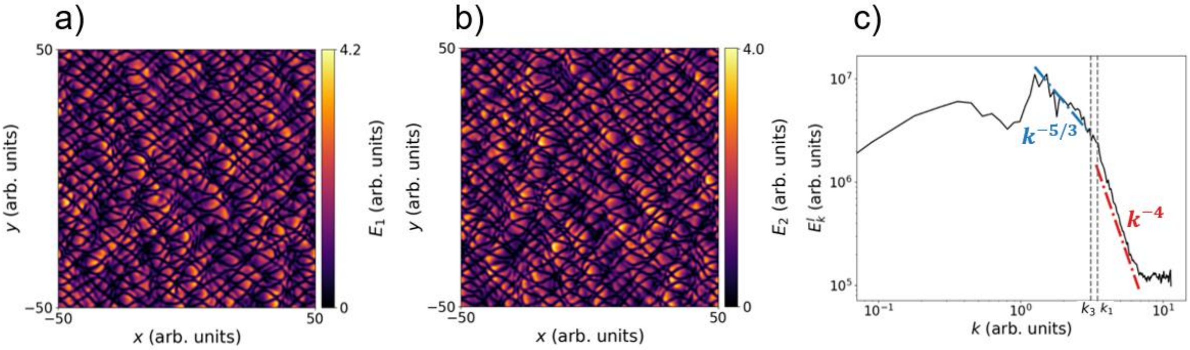

Figure 2: Numerical simulation of the two-fluid method for $v = 3.46c_s$, corresponding to the dispersion relation plotted in Figure 1-c). a) and b) show the density profiles of the two components. c) show the incompressible component of the kinetic energy and the corresponding scaling laws. These results were obtained with $\nu = 1$, $q = 2$ and $\rho = 1$

For exploring the second method of generating turbulent states, through the decay of dark soliton stripes, the light beam E_2 is turned off, and only the light beam E_1 propagates. This consists in a uniform background disturbed by a small amplitude white noise with a set of dark soliton stripes, as depicted in Figure 3-a). During the propagation of this state inside the NLC cell, the dark soliton stripes decay into vortex pairs and generate a turbulent state, as plotted in Figure 3-b). In Figure 3-c), the incompressible component of the kinetic energy is plotted, where two energy cascades are identified: an energy cascade with a $k^{-5/3}$ power law, as already seen in the two fluids of light method; and a different one that follows a k^{-3} power law, which is associated with the structure of the vortices present in the turbulent state (Bradley and Anderson 2012).

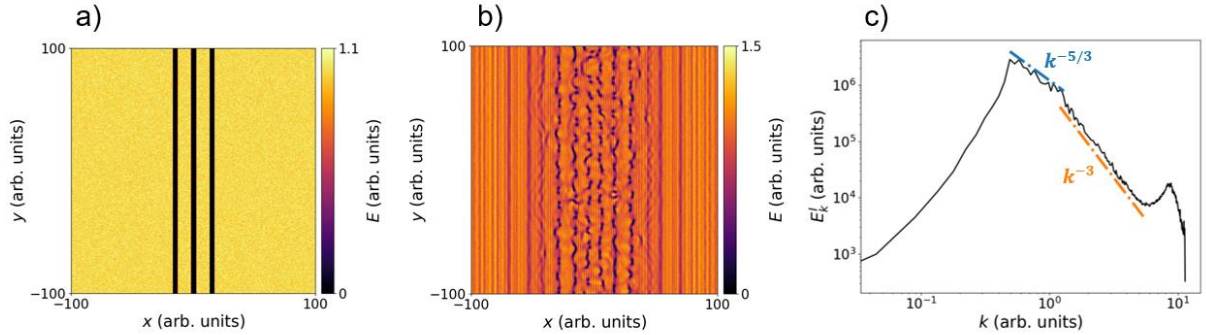

Figure 3: Numerical simulation of the decay of dark soliton stripes method. a) corresponds to the initial state and b) to the state after propagating inside the nematic liquid crystal cell. c) show the incompressible component of the kinetic energy and the corresponding scaling laws. These results were obtained with $\nu = 1$, $q = 2$ and $\rho = 1$

The observation of these different power laws in the incompressible component of the kinetic energy in both methods suggests that we are in the presence of turbulent states. Thus, these methods offer an interesting playground to explore, in a tabletop experiment and under controlled conditions, the generation of turbulent regimes. Furthermore, they allow exploring the origin of the different power laws and their relationship with the structures that can form in the fluid. Now, it is important to explore the different range of parameters allowed by the NLCs and how these parameters influence and can be used to optimize the formation of these turbulent states. In particular, it is important to understand the effect of nonlocality in the formation of these regimes. However, these studies are out of the scope of the present work, and we leave them for future studies.

4. Conclusions

Nematic liquid crystals are an interesting optical material for creating fluids of light and implement optical analogues. In this work, we reported the preliminary findings on the use of these systems for exciting different turbulent regimes with a two-fluid model. The dispersion relation was studied, and it was demonstrated that different energy injection ranges can be achieved by changing the velocity of one of the fluids. These different ranges of energy injection are related to different turbulent configurations. We then simulated a configuration with a specific energy injection range, and by studying the incompressible component of the kinetic energy, we observed the energy cascades that characterize these turbulent regimes. We also reported the preliminary findings regarding the creation of turbulent states through decay of dark soliton stripes. With this method, it was also possible to observe energy cascades in the incompressible component of the kinetic energy that are characteristic of turbulent regimes.

Nematic liquid crystals stand thus as an interesting playground for creating and exploring optical analogues of turbulent regimes, which can be performed through different methods. Their tunable properties can help generate these states, and their experimental realization can give new insights to the understanding of turbulent phenomena.

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